

Application of LQ-based semi-active suspension control in a vehicle



Andreas Unger^{b,*}, Frank Schimmack^b, Boris Lohmann^a, Ralf Schwarz^b

^a Institute of Automatic Control, Technische Universität München, 85748 Garching, Germany

^b AUDI AG, 85049 Ingolstadt, Germany

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ABSTRACT

A linear quadratic (LQ) based controller and observer concept for a semi-active full-car model is implemented and compared to a skyhook controller using a real vehicle in this paper. Especially an observer including a new modified road model is presented, which reliably filters low frequency disturbances induced by ascending or descending steep hills. Furthermore, methods for the parametrization of the quadratic cost function are presented and it is shown in experiments for different vehicle masses that ride comfort and road holding can be significantly improved by using semi-active suspension control. In order to take nonlinear component characteristics or suspension friction into account a parametrized nonlinear full-car model is presented, which is used for the determination of the controller cost function weights. The performance evaluation is done using a 4-poster test rig as well as measurements conducted on a real road.

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1. Introduction

The main tasks of a vehicle suspension system are to provide a high level of ride comfort by isolating the chassis mass from road disturbances and to improve road holding by preventing the wheel from losing road contact. Several studies have shown that this conflict can be eased by using controlled suspension systems instead of passive ones. Especially semi-active dampers are widespread in industrial application because of their performance increase compared to passive suspensions and their lower energy consumption compared to active systems (Guglielmino, Sireteanu, Stammers, Ghita, & Giuclea, 2008).

Methods for controlling semi-active suspension systems reach back to the works of Karnopp (Karnopp, 1983; Karnopp, Crosby, & Harwood, 1974; Karnopp & Heess, 1991) who has proposed the skyhook algorithm in order to improve the ride comfort. Several extensions of skyhook control have been suggested in the past decades (Ahmadian, Song, & Southward, 2004; Guo, Xu, Xing, & Pan, 2006; Hong, Sohn, & Hedrick, 2002; Rao & Narayanan, 2008; Savaresi & Spelta, 2009; Shamsi & Choupani, 2008) and especially the combination of a skyhook-based controller with a groundhook algorithm promises additional improvements on ride safety (Hudha, Jamaluddin, Samin, & Rahman, 2005; Valasek & Kortüm, 2000; Yi & Song, 1999).

As the skyhook algorithm seems well studied, it is not surprising that this concept is mainly used if semi-active suspension controllers are implemented in real vehicles. Examples where skyhook control has been investigated experimentally can be found in Venhovens (1994) for a Volvo 480, in Lauwerys, Swevers, and Sas (2005) for an Audi A6, in Niemz (2006) for an Opel Astra or in Sankaranarayanan et al. (2008), where a Ford Transit Connect has been tested with four independent skyhook/groundhook controller combinations. Also the control concepts of series production vehicles are based on skyhook control, as it can be found for Audi (Schöpfel, Stingl, Schwarz, Dick, & Biesalski, 2007; Schwarz, Biesalski, Schöpfel, & Stingl, 2008), BMW (Jautze, Bogner, Eggendinger, Rekowitz, & Stumm, 2008) or Mercedes-Benz (Früh et al., 2009).

Even though it has been shown that clipped optimal control offers performance benefits for the control of semi-active suspensions (Butsuen, 1989; Gordon & Best, 1994; Unger, Koch, & Lohmann, 2011) only quite few results on the experimental validation of model-based control techniques in a real vehicle are known in the literature so far. For the suspension control using a semi-active full-car model only some simulation studies exist (Jahromi & Zabihollah, 2010; Lu & DePoyster, 2002).

State observers for vehicle suspension systems have also been investigated. In Koch, Kloiber, Pellegrini, and Lohmann (2010) a Kalman filter-based observer structure has been proposed which takes the nonlinearities of the passive damper into account. An experimental investigation of an H_∞ -observer has been made in Aubouet, Dugard, and Senane (2010) and some studies on the robustness of state estimation can be found in Fröhlich (2007).

This paper extends the results presented in Unger, Schimmack, Lohmann, and Schwarz (2011) where the experimental evaluation of

* Corresponding author. Tel.: +49 841 89574438.

E-mail addresses: andreas.unger@audi.de (A. Unger), frank.schimmack@audi.de (F. Schimmack), lohmann@tum.de (B. Lohmann), ralf.schwarz@audi.de (R. Schwarz).

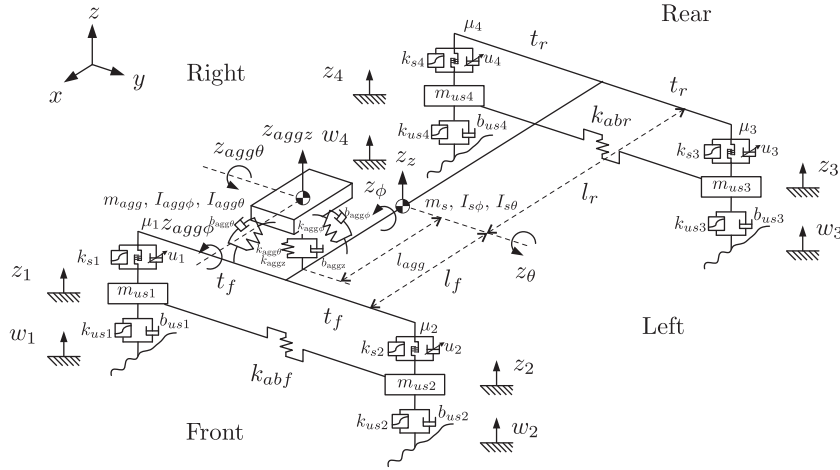


Fig. 1. The nonlinear full-car model.

an LQ-based semi-active estimator and controller concept in a real vehicle has been investigated. The parametrization of the quadratic cost function is addressed and two different controllers are presented. One controller is optimized to improve ride comfort considering a comfort measure proposed by Hennecke (1995) and the other controller is optimized for reducing the braking distance. Additionally to Unger, Schimmack, et al. (2011) a new modified road model is presented, which is used for the observer design in order to compensate for low frequency disturbances induced by ascending or descending steep hills. Measurements taken on a real street and the validation of the observer on a 4-poster test rig prove the high estimation quality of the concept also in challenging situations. Furthermore, it is shown by measurements that both controllers outperform the passive suspension and a skyhook-based controller in their respective aims, even for an increased vehicle mass. A full-car model, which is parametrized by measurements on a 4-poster test rig, is used for the observer and controller design. Additionally, an identified nonlinear full-car model is presented, which is used for the optimization routines to parametrize the controller.

The remainder of the paper is organized as follows: In Section 2 the nonlinear and the linearized full-car models are presented. The structure of the controller and the parametrization are described in Section 3, whereas the observer including the modified road model is content of Section 4. The experimental results are discussed in Section 5 before the paper is finally concluded in Section 6.

2. The modeling of the system

In order to apply model-based control techniques to a real vehicle, a model is required for the simulation, the controller and observer design and the optimization of the cost function weights. A nonlinear full-car model, which is presented for purposes of simulation and optimization, is linearized and reduced to seven degrees of freedom for the observer and controller design. The road disturbance can be described by four first order shape filters for each wheel which are joined with the linear full-car model.

2.1. The nonlinear full-car model

The vertical dynamics of a vehicle can be described by a full-car model (Chalasani, 1986), which is shown in Fig. 1. It consists of four wheel masses m_{us} , each with one vertical degree of freedom, the chassis mass (mass m_s , moments of inertia $I_{s\phi}, I_{s\theta}$), with freedoms of motion in the vertical, roll and pitch directions and

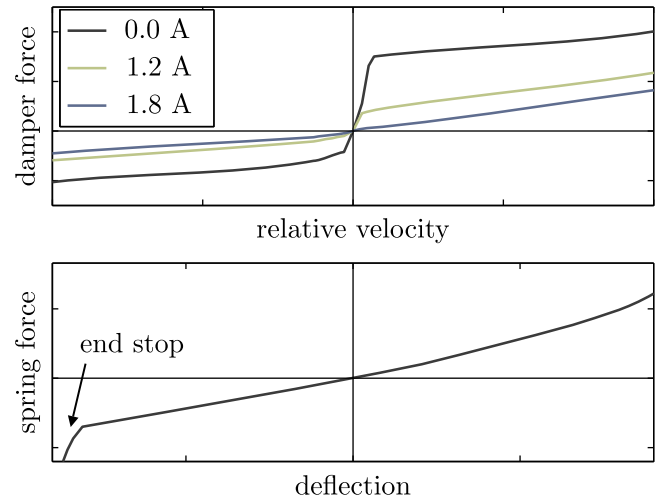


Fig. 2. The characteristics of the semi-active damper $u(\mathbf{x}, \lambda)$ and the spring $f_{spring}(\mathbf{x})$ (scaling omitted).

finally the mass of the aggregate¹ (mass m_{agg} , moments of inertia $I_{agg\phi}, I_{agg\theta}$) also with three degrees of freedom in the vertical, roll and pitch directions. The vehicle to be considered in this work is equipped with steel springs and four semi-active dampers. The springs (spring constant k_s) have a typical progressive characteristic including an end stop which has been directly measured and can be seen in Fig. 2. The dampers are of hydraulic double-tube type with an internal controllable solenoid valve. By applying a current λ between 0 A and 1.8 A to the valve, the damping force can be modified between a minimum and a maximum bound as shown in Fig. 2. The tire (spring constant k_{us} and damping rate b_{us}) can be considered as an air spring (Mitschke & Wallentowitz, 2004) and therefore has a progressive characteristic. Moreover, friction effects (friction factor μ) in the suspension are considered using an arctangent function. All other elements are calculated linearly with the respective elements shown in Fig. 1. The equations of motion can be summarized using the suspension force $F_{s,i} = f_{spring,i}(\mathbf{x}) + f_{friction,i}(\mathbf{x})$ ($i \in [1..4]$), the tire force $F_{us,i}$, the forces F_{abr}, F_{abf} generated by the rear and front anti-roll bars (stiffnesses k_{abr}, k_{abf}) and the forces and torques of the aggregate $F_{aggz}, T_{agg\phi}, T_{agg\theta}$. Furthermore, the forces of the semi-active

¹ The components engine and transmission are denoted as aggregate.

damper $\mathbf{u}(\mathbf{x}, \lambda)$, which are used as control inputs of the system and the state vector $\mathbf{x} = [z_1 \dot{z}_1 z_2 \dot{z}_2 \dots z_{\text{agg}\theta} \dot{z}_{\text{agg}\theta}]^T$ are defined.

This leads to the dynamic equations for the wheels

$$m_{us1}\ddot{z}_1 = F_{us1} - F_{s1} - F_{abf} - u_1, \quad (1)$$

$$m_{us2}\ddot{z}_2 = F_{us2} - F_{s2} + F_{abf} - u_2, \quad (2)$$

$$m_{us3}\ddot{z}_3 = F_{us3} - F_{s3} + F_{abr} - u_3, \quad (3)$$

$$m_{us4}\ddot{z}_4 = F_{us4} - F_{s4} - F_{abr} - u_4, \quad (4)$$

the body

$$m_s\ddot{z} = F_{s1} + F_{s2} + F_{s3} + F_{s4} - F_{aggz} + u_1 + u_2 + u_3 + u_4, \quad (5)$$

$$I_{s\phi}\ddot{\phi} = (-F_{s1} - u_1 + F_{s2} + u_2 - 2F_{abf})l_f + (F_{s3} + u_3 - F_{s4} - u_4 - 2F_{abr})l_r - T_{agg\phi}, \quad (6)$$

$$I_{s\theta}\ddot{\theta} = (-F_{s1} - u_1 - F_{s2} - u_2)l_f + (F_{s3} + u_3 + F_{s4} + u_4)l_r + F_{aggz}l_{agg} - T_{agg\theta}, \quad (7)$$

and the aggregate

$$m_{agg}\ddot{z}_{aggz} = F_{aggz}, \quad (8)$$

$$J_{agg\phi}\ddot{\phi}_{agg\phi} = T_{agg\phi}, \quad (9)$$

$$J_{agg\theta}\ddot{\theta}_{agg\theta} = T_{agg\theta}. \quad (10)$$

2.2. Linearization and order reduction

As a linear model is used for the observer and the controller design, the nonlinear model of the previous section is linearized by substituting the nonlinear elements of the model by linear ones. According to Mitschke and Wallentowitz (2004) and the parameters of the aggregate, it can be shown that the aggregate mainly influences the body movement at frequencies greater than 9 Hz. In ISO (1997) it is remarked that the relevant frequency range for ride comfort is up to 8 Hz and therefore the degrees of freedom for the aggregate are neglected for the linear model. The equations of motion can be summarized using the disturbance vector $\omega = [w_1 \dots w_4]^T$ and the matrices $\mathbf{A} \in \mathbb{R}^{(14 \times 14)}$, $\mathbf{B}_u \in \mathbb{R}^{(14 \times 4)}$ and $\mathbf{B}_\omega \in \mathbb{R}^{(14 \times 4)}$ to the state space notation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_u\mathbf{u} + \mathbf{B}_\omega\omega. \quad (11)$$

The measured outputs $\mathbf{y}$ are, according to the sensor configuration of the vehicle, the four relative suspension velocities, the heave body acceleration and the roll and pitch rates. The output equation can be calculated by using the matrices $\mathbf{C}_y \in \mathbb{R}^{(7 \times 14)}$, $\mathbf{D}_{yu} \in \mathbb{R}^{(7 \times 4)}$ and $\mathbf{D}_{y\omega} \in \mathbb{R}^{(7 \times 4)}$ to

$$\mathbf{y} = \mathbf{C}_y\mathbf{x} + \mathbf{D}_{yu}\mathbf{u} + \mathbf{D}_{y\omega}\omega + \nu \quad (12)$$

where ν denotes the measurement noise, which is needed for the observer design described in Section 3. The parameter values of the linear elements are determined by optimization, as it is described in Section 2.4.

For the LQ-based controller design the penalized variables are the vertical acceleration $\ddot{z}$, the roll acceleration $\ddot{\phi}$ and the pitch acceleration $\ddot{\theta}$ of the body as well as the tire deflections $\Delta z_{ws,i} = z_i - \omega_i$ by means of the scalar weighting factors ρ_z , ρ_ϕ and ρ_θ . By using the matrices $\mathbf{C}_w \in \mathbb{R}^{(7 \times 14)}$, $\mathbf{D}_{wu} \in \mathbb{R}^{(7 \times 4)}$ and $\mathbf{D}_{w\omega} \in \mathbb{R}^{(7 \times 4)}$ the equation for calculating the above mentioned outputs

$$\mathbf{w} = \mathbf{C}_w\mathbf{x} + \mathbf{D}_{wu}\mathbf{u} + \mathbf{D}_{w\omega}\omega \quad (13)$$

can be introduced.

2.3. Road model

In ISO (1995) it is reported that the power spectral density (PSD) of the road displacement can be approximated by

$$S_w(f) = \frac{1}{v} A \left(\frac{2\pi f}{v} \right)^n \quad (14)$$

with the vehicle velocity v and the roughness factor of the road A in [m]. As described in ISO (1995) typical roads can be classified using the scalar factor A , whereby higher values of A indicate rougher roads. The measurements in ISO (1995) show that the exponent n is independent from the road class, whereas the approximation of $n = -2$ holds in general. By incorporating that the amplitudes of very low frequencies $f \rightarrow 0$ stay finite and considering $n = -2$, Eq. (14) results in a first order shape filter

$$\dot{\omega}(t) = -\beta v \omega(t) + v \xi \quad (15)$$

where βv is the cut-off frequency and ξ is a gaussian distributed white noise input with the spectral density A (Gaspar & Nadai, 2007; Hrovat, 1997; Koch, 2011; Venhovens, 1993). The parameters are chosen according to Venhovens (1993) to $\beta = 0.2$, $v = 30$ m/s and A results from the optimization routine of the Kalman filter.

By increasing the filter order of the shape filter, the approximation of the PSD can be improved especially at high frequencies, like it has been investigated in Türkay and Akcay (2005). But as the improvements are only slight and an increased filter order leads to higher computational effort in real-time control, the first order approximation of Eq. (15) is considered for the observer design in Section 4.

In order to augment the system equations with the road model, Eq. (15) is extended to all four wheels

$$\dot{\omega} = \underbrace{[-\beta v, -\beta v, -\beta v, -\beta v] \mathbf{I}_4}_{\mathbf{A}_s} \omega + \underbrace{[v, v, v, v] \mathbf{I}_4}_{\mathbf{B}_s} \xi \quad (16)$$

which is then appended to the state space representation leading to

$$\underbrace{\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\omega} \end{bmatrix}}_{\dot{\mathbf{x}}_{\text{ext}}} = \underbrace{\begin{bmatrix} \mathbf{A} & \mathbf{B}_w \\ \mathbf{0} & \mathbf{A}_s \end{bmatrix}}_{\mathbf{A}_{\text{ext}}} \underbrace{\begin{bmatrix} \mathbf{x} \\ \omega \end{bmatrix}}_{\mathbf{x}_{\text{ext}}} + \underbrace{\begin{bmatrix} \mathbf{B}_u \\ \mathbf{0} \end{bmatrix}}_{\mathbf{B}_{u,\text{ext}}} \mathbf{u} + \underbrace{\begin{bmatrix} \mathbf{0} \\ \mathbf{B}_s \end{bmatrix}}_{\mathbf{B}_{\xi,\text{ext}}} \xi, \quad (17)$$

$$\mathbf{y} = \underbrace{[\mathbf{C}_y \ \mathbf{D}_{y\omega}]}_{\mathbf{C}_{y,\text{ext}}} \underbrace{\begin{bmatrix} \mathbf{x} \\ \omega \end{bmatrix}}_{\mathbf{x}_{\text{ext}}} + \underbrace{\mathbf{D}_{yu}}_{\mathbf{D}_{yu,\text{ext}}} \mathbf{u} + \nu, \quad (18)$$

$$\mathbf{w} = \underbrace{[\mathbf{C}_w \ \mathbf{D}_{w\omega}]}_{\mathbf{C}_{w,\text{ext}}} \underbrace{\begin{bmatrix} \mathbf{x} \\ \omega \end{bmatrix}}_{\mathbf{x}_{\text{ext}}} + \underbrace{\mathbf{D}_{wu}}_{\mathbf{D}_{wu,\text{ext}}} \mathbf{u}. \quad (19)$$



Fig. 3. Experimental vehicle on the 4-poster test rig.

2.4. Parameter identification

It is assumed that the vehicle is symmetric along its longitudinal axis that the geometry is known and that the spring and damper characteristics are determined by separate component tests. The remaining parameters of the linear and nonlinear full-car models have been identified by measurements of the experimental vehicle on a 4-poster test rig (Fig. 3). Sinus sweeps between 0.5 Hz and 25 Hz for exciting the heave, pitch, roll and warp movements have been used as input signals for the identification. As still the amount of unknown parameters is too large for global optimization, the parameters are combined to groups, which are identified separately using a Trust-Region-Reflective algorithm provided by the MATLAB optimization toolbox (Coleman & Li, 1996). The procedure is repeated iteratively until the variation of a global fitness value has stagnated. A comparison between the measured vertical acceleration and the simulation result is given in Section 5.2.

3. Semi-active suspension control

For the suspension control of the full-car, four semi-active damping devices are used. Accordingly, the damping force can be adjusted by a controllable valve, but it cannot exceed the minimum or maximum bounds which are shown in Fig. 2. As most semi-active control laws do not explicitly account for these constraints, the demanded forces $\mathbf{u}_{\text{des}}$ must be clipped to match the controllability range of the damper, i.e.

$$\mathbf{u} = \text{sat}(\mathbf{u}_{\text{des}}). \quad (20)$$

Since the system is dissipative for any force generated by the semi-active damper, the overall system remains stable despite the control input is saturated. However, the performance may not be optimal for the semi-active case any more. This issue has been addressed by Butsuen (1989), where the optimal control law for a semi-active suspension system has been derived considering a quadratic performance index. Using the calculus of variations, Euler's equation and Lagrange multipliers in order to consider the state dependent constraints, the optimal control law for semi-active quarter-car suspensions has been proven to be the same as clipped optimal control, which is introduced in Section 3.2. For a semi-active full-car model a difference between the optimal control law and clipped optimal control can be observed, but the differences are negligible (Butsuen, 1989).

Also the generalized benchmark of semi-active suspension controllers in Unger, Koch, et al. (2011) has been shown in experiments that clipped optimal control is a reasonable choice regarding computational effort and performance for real-time application. Therefore, clipped optimal control is applied for the semi-active suspension control in this work. A simulative benchmark of semi-active control techniques considering for instance also LPV or H_∞ control can be found in Poussot-Vassal, Spelta, Senname, Savaresi, and Dugard (2011).

In order to benchmark the clipped optimal control scheme, a skyhook controller is used as a reference. In the following section, the skyhook algorithm for a full-car model is introduced, then clipped optimal control will be presented.

3.1. Skyhook control

The main idea of skyhook control, introduced by Karnopp et al. (1974), is to damp the sprung mass with respect to an inertial reference frame. In the case of a full-car model also the pitch and roll movements are damped inertially by applying a damping factor on the pitch and roll rates. The desired forces, which have to

be clipped according to (20), can be calculated as

$$\mathbf{u}_{\text{sky}} = - \underbrace{\begin{bmatrix} \frac{l_r}{2(l_r+l_f)} & -\frac{1}{4t_f} & -\frac{1}{2(l_r+l_f)} \\ \frac{l_r}{2(l_r+l_f)} & \frac{1}{4t_f} & -\frac{1}{2(l_r+l_f)} \\ \frac{l_f}{2(l_r+l_f)} & \frac{1}{4t_r} & \frac{1}{2(l_r+l_f)} \\ \frac{l_f}{2(l_r+l_f)} & -\frac{1}{4t_r} & \frac{1}{2(l_r+l_f)} \end{bmatrix}}_{\mathbf{T}_{\text{mc}}} \begin{bmatrix} b_{\text{sky},z} & 0 & 0 \\ 0 & b_{\text{sky},\phi} & 0 \\ 0 & 0 & b_{\text{sky},\theta} \end{bmatrix} \begin{bmatrix} \dot{z}_z \\ \dot{z}_\phi \\ \dot{z}_\theta \end{bmatrix} \quad (21)$$

using the geometric transformation matrix $\mathbf{T}_{\text{mc}}$.

3.2. Clipped optimal control

Clipped optimal control (coc) is a method where state feedback control can be used for the control of semi-active suspensions. First, a linear quadratic regulator is designed with regard to the fully active suspension system (a suspension system with a high-bandwidth actuator incorporated in parallel to the main spring and damper, see, e.g. Mitschke & Wallentowitz, 2004). Subsequently, the desired forces are clipped to the controllability range of the semi-active damper. A state feedback law

$$\bar{\mathbf{u}}_{\text{coc}} = -\mathbf{K}_{\text{coc}}\mathbf{x} \quad (22)$$

is designed for the system described by (11)–(13) which minimizes the cost function

$$J = \int_0^\infty (\mathbf{w}^T(\tau)\mathbf{Q}_w\mathbf{w}(\tau) + \mathbf{u}^T(\tau)\mathbf{R}_w\mathbf{u}(\tau)) d\tau \quad (23)$$

with

$$\mathbf{Q}_w = \begin{bmatrix} \rho_{rs}\mathbf{I}_4 & \mathbf{0} \\ \mathbf{0} & \rho_a & 0 & 0 \\ \mathbf{0} & 0 & \rho_\theta & 0 \\ \mathbf{0} & 0 & 0 & \rho_\phi \end{bmatrix}, \quad \mathbf{R}_w = \rho_u\mathbf{I}_4. \quad (24)$$

The feedback matrix $\mathbf{K}_{\text{coc}}$ is calculated by solving the Riccati equation (Anderson & Moore, 2007)

$$\mathbf{A}^T\mathbf{P}_c + \mathbf{P}_c\mathbf{A} - (\mathbf{P}_c\mathbf{B}_u + \mathbf{N}_c)\mathbf{R}_c^{-1}(\mathbf{B}_u^T\mathbf{P}_c + \mathbf{N}_c^T) + \mathbf{Q}_c = \mathbf{0} \quad (25)$$

with $\mathbf{P}_c = \mathbf{P}_c^T > 0$ and

$$\begin{aligned} \mathbf{Q}_c &= \mathbf{C}_w^T\mathbf{Q}_w\mathbf{C}_w, \\ \mathbf{R}_c &= \mathbf{D}_{wu}^T\mathbf{Q}_w\mathbf{D}_{wu} + \mathbf{R}_w \\ \mathbf{N}_c &= \mathbf{C}_w^T\mathbf{Q}_w\mathbf{D}_{wu}. \end{aligned} \quad (26)$$

The state feedback matrix $\mathbf{K}_{\text{coc}} \in \mathbb{R}^{(4 \times 14)}$ is obtained by solving

$$\mathbf{K}_{\text{coc}} = \mathbf{R}_c^{-1}(\mathbf{B}_u^T\mathbf{P}_c + \mathbf{N}_c^T). \quad (27)$$

The force which is applied by the semi-active damper must be clipped analog to Eq. (20). By using the damper characteristic (Fig. 2) the resulting control force $\mathbf{u}$ is then transformed into a current, which is applied to the valve.

The result is a linear control law, which is implemented in the experimental vehicle described by the nonlinear full-car model presented in Section 2.1. The nonlinearities of the system are the damping force, the spring force, the tire force and friction effects. The main nonlinearity is the damping force, which is used as control input for the control design, whereby the effect of saturation has already been discussed. By looking at Fig. 2, it can be observed that the spring force, as the second important nonlinearity, can be linearized with marginal loss of precision when the suspension operates within its limits. As the remaining nonlinearities only have minor influence on the system behavior, no further adjustments are made for applying the linear control law to the real system.

3.3. Determining the cost function weights

It has been shown in Unger, Schimmack, et al. (2011) and Do, Sename, Dugard, and Soualmi (2011) that the parameters of the quadratic cost function (23) can be determined by minimizing more complex performance criteria using genetic algorithms. Thereby, two performance measures are introduced, one for ride comfort and one for ride safety, which are evaluated using the nonlinear full-car model. Then two controller parametrizations are optimized in order to minimize the respective measures.

For the design of the comfort orientated controller the comfort index proposed by Hennecke (1995)

$$\Pi_{\text{comf}} = \sqrt{k_1 \left(\frac{1}{t_e} \int_0^{t_e} a_{w,\phi}^4(t) dt \right)^{2/4} + k_2 \left(\frac{1}{t_e} \int_0^{t_e} a_{w,z}^4(t) dt \right)^{2/4}} \quad (28)$$

is used including the frequency weighted heave ($a_{w,z}$) and pitch ($a_{w,\phi}$) accelerations of the body according to ISO (1997) and weighting factors k_1 , k_2 for stochastic vibrations in the time interval $[0; t_e]$. It is noted that despite the frequency range between 4 and 8 Hz is emphasized by the weighting filter, the other frequencies, e.g. the range between 1 and 2 Hz which makes passengers feel sick, are also taken into account.

In order to design a controller which improves the ride safety, the braking distance from 85 kph to a full stop on a rough road is used as a safety index. The results presented in Unger, Schimmack, et al. (2011) have shown in experiments that the braking distance can be reduced by 0.3 m compared to the passive hard setting and 1.8 m compared to a lightly damped suspension setting for a luxury car. The procedure incorporates the extension of the nonlinear full-car model by a nonlinear, longitudinal tire model proposed by Pacejka (2005) which describes the coherence of F_z , the longitudinal tire force F_x and the wheel slip κ , i.e.

$$F_x = (D \sin(C \arctan(BX - E(BX - \arctan(BX)))))) + S_v \quad (29)$$

with $X = \kappa + S_h$, $S_h = S_h(F_z)$, $S_v = S_v(F_z)$, $D = D(F_z)$ and $B = B(F_z)$. For the deceleration of the car the maximum possible force $F_{x,\text{max},i}$ is taken, which is equal to the sum $D + S_v$ in (29) and the traveled distance Π_{safe} in [m] until the vehicle comes to a stop is evaluated.

The tuning of the controllers is accomplished using numerical optimization of the unique diagonal elements of $\mathbf{Q}_{w,i} \in \mathbb{R}^{(7 \times 7)}$ and $\mathbf{R}_{w,i} \in \mathbb{R}^{(4 \times 4)}$ in (23). Thereby, in comparison with a direct numerical optimization of the matrix elements of $\mathbf{K}_{\text{coc}}$, the number of optimization variables is reduced from 56 to 5 by using the proposed LQ-based design approach. The objective functions are $\Pi_{\text{comf}}(\mathbf{Q}_{w,1}, \mathbf{R}_{w,1})$ for the comfort orientated controller and $\Pi_{\text{safe}}(\mathbf{Q}_{w,2}, \mathbf{R}_{w,2})$ for the safety orientated controller respectively. For each function call, a closed-loop simulation of the nonlinear full-car model is carried out, whereby a measured country road profile is employed as disturbance. The optimization problem is solved using the genetic algorithm NSGA-II described in Deb, Pratap, Agarwal, and Meyarivan (2002) with the number of generations set to 150 and a population size of 250. The evaluation of the 37 500 function calls takes approximately 15 h computational time on a Core i7 2.8 GHz CPU for each controller setting. This clearly indicates that the reduction of the optimization variables has been a necessary step.

4. State observer design

The application of the full state feedback law presented in the previous section to a real vehicle requires the knowledge of all 14 states of the full-car model. As the direct measurement of all states is either technically hardly practicable or too expensive the required signals have to be estimated. The sensor configuration of the vehicle allows a filter-based estimation of the remaining

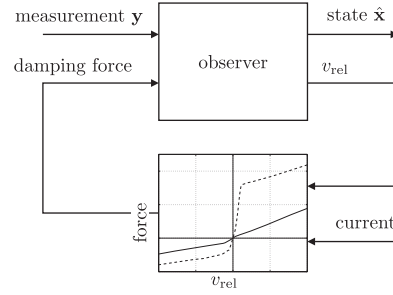


Fig. 4. Structure of the observer.

states, but as it has already been investigated in Koch, Kloiber, and Lohmann (2010), an observer-based concept provides superior performance, when the estimation of the whole state vector is considered. Furthermore, a model-based observer allows the compensation of noise and disturbances on the sensors without introducing phase distortions. Because of the stochastic nature of the sensor noise and the road disturbance, the application of a Kalman filter algorithm is a reasonable choice.

4.1. Kalman filter

In order to consider the nonlinear damping force, which depends on the relative velocity between the chassis and the wheel mass and on the current applied to the valve, an observer structure is employed, which has been proposed by Ohsaku (2000) and Koch, Kloiber, Pellegrini, et al. (2010). As shown in Fig. 4, the nonlinear damping force is calculated externally and is considered as a known force input. Therefore, a linear observer structure can be used and the linear passive damping b_{si} is set to 0.

As it has been pointed out in Section 2.3 the disturbance input of the road model ξ is zero-mean and gaussian. The measurement noise ν is acting on the output equation and is also assumed to be zero-mean. Both inputs are uncorrelated satisfying

$$\mathbf{E}(\xi\xi^T) = \mathbf{Q}_\xi, \quad \mathbf{E}(\nu\nu^T) = \mathbf{R}_\nu, \quad \mathbf{E}(\xi\nu^T) = \mathbf{0}. \quad (30)$$

According to Kwakernaak and Sivan (1972) the observer equation for the extended state space equations (17)–(19) which minimizes the error between the estimated state $\hat{\mathbf{x}}_{\text{ext}}$ and the real state $\mathbf{x}_{\text{ext}}$ is

$$\dot{\hat{\mathbf{x}}}_{\text{ext}} = (\mathbf{A}_{\text{ext}} - \mathbf{L}\mathbf{C}_{y,\text{ext}})\hat{\mathbf{x}}_{\text{ext}} + (\mathbf{B}_{u,\text{ext}} - \mathbf{L}\mathbf{D}_{yu,\text{ext}})\mathbf{u} + \mathbf{L}\mathbf{y} \quad (31)$$

with the optimal gain

$$\mathbf{L} = \mathbf{P}_0 \mathbf{C}_{y,\text{ext}}^T \mathbf{R}_\nu^{-1} \quad (32)$$

and $\mathbf{P}_0$ being the solution of the Riccati equation

$$\mathbf{A}_{\text{ext}}\mathbf{P}_0 + \mathbf{P}_0\mathbf{A}_{\text{ext}}^T - \mathbf{P}_0\mathbf{C}_{y,\text{ext}}^T \mathbf{R}_\nu^{-1} \mathbf{C}_{y,\text{ext}}\mathbf{P}_0 + \mathbf{Q}_0 = \mathbf{0} \quad (33)$$

with $\mathbf{Q}_0 = \mathbf{B}_{\xi,\text{ext}}\mathbf{Q}_\xi\mathbf{B}_{\xi,\text{ext}}^T$ and $\mathbf{P}_0 = \mathbf{P}_0^T > \mathbf{0}$.

4.2. Low frequency disturbance compensation

In order to tune the Kalman filter, the diagonal elements of the matrices

$$\mathbf{Q}_\xi = q_\xi \mathbf{I}_4; \quad \mathbf{R}_\nu = \begin{bmatrix} r_1 \mathbf{I}_4 & \mathbf{0} \\ \mathbf{0} & \begin{bmatrix} r_2 & 0 & 0 \\ 0 & r_3 & 0 \\ 0 & 0 & r_4 \end{bmatrix} \end{bmatrix} \quad (34)$$

have to be adjusted, containing the scalar weighting factors q_ξ , r_1 , r_2 , r_3 and r_4 . This can be either done iteratively (e.g. Fröhlich, 2005; Venhovens, 1993) or by means of optimization (e.g. Koch, Kloiber, Pellegrini, et al., 2010; Unger, Schimmack, et al., 2011) using reference data generated by a detailed simulation model and a zero-mean road

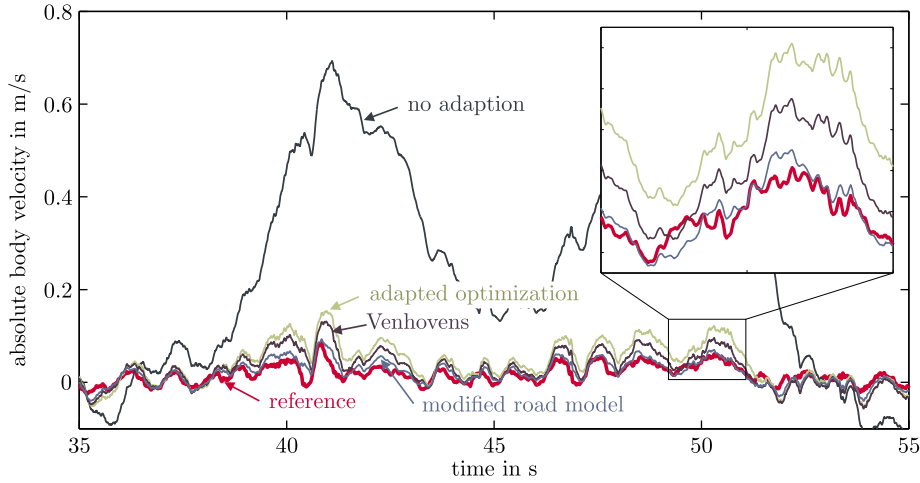


Fig. 5. Absolute body velocity while ascending a road of 18% slope.

profile as input. As already observed by Venhovens (1993) a well tuned Kalman filter in simulation is prone to estimate low frequency disturbances which occur on real streets by leaning road surfaces, ascending or descending hills or steady-state cornering. Fig. 5 shows the measured absolute body velocity while ascending a hill of 18% slope with the experimental vehicle shown in Fig. 3 for an observer laid out by the above mentioned procedure. The reference velocity has been generated by integration of the vertical body acceleration and highpass filtering with zero-phase using MATLAB's function `filtfilt` for eliminating the low frequency disturbance. It can be seen that the estimation of the Kalman filter is considerably corrupted and cannot be used for suspension control when no adaption is made.

In order to get rid of the low frequency disturbance in the estimation, it might be possible to apply a highpass filter to the sensor signals or the estimated variables of the Kalman filter. The cut-off frequency of the filter has been selected by evaluating measurements taken on several different roads. The results show that a reliable filtering of the disturbances requires a minimal cut-off frequency of 0.8 Hz, what is in accordance to the value of approximately 0.5 Hz reported in Venhovens (1993). However, a cut-off frequency of 0.8 Hz causes a considerable phase distortion at the body's natural frequency that leads to counteracting control actions in semi-active suspension control.

A large improvement can already be achieved by adapting the optimization routine and considering reference data measured while driving on a real street including hills instead of using reference signals generated by simulation. The reference data can be obtained from the internal sensors by integration and geometric considerations using highpass filtering with zero-phase. The optimization follows the procedure described in Section 3.2 for tuning the controllers. The five optimization variables are optimized by using the NSGA-II-algorithm with 150 generations and a population size of 250. The improvement of the estimation using the adapted optimization is shown in Fig. 5 where it can be seen that large parts of the low frequency disturbance are eliminated.

Additional benefit can be gained by assuming that the measurement noise is not white noise but can be modeled by

$$\dot{\nu} = \mu \quad (35)$$

where μ is white noise (Venhovens, 1993). This leads to a Kalman filter with an internal proportional-integral feedback and Venhovens has shown that the low frequency drift in the absolute velocity can be eliminated for a double-lane change maneuver in simulation. As depicted in Fig. 5 the estimation quality is improved

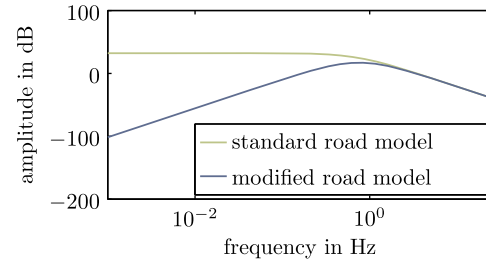


Fig. 6. Transfer functions of the road models.

by using the approach of Venhovens, but some disturbance effects still remain.

The solution proposed in this work uses an extension of the road model introduced in Section 2.3 by a highpass filter with a cut-off frequency of $f_{hp} = 0.8$ Hz. Therefore, only high frequency road inputs are assumed by the road model and zero mean estimates are enforced. In order to obtain the modified road model, the standard road model (15) in frequency domain

$$G_{lp} = \frac{v_{Fzg}}{s + \beta v_{Fzg}}$$

is extended by a highpass filter

$$G_{hp} = \frac{T_{hp}s}{T_{hp}s + 1} \quad (36)$$

where $T_{hp} = 1/2\pi f_{hp}$ and 's' denotes the Laplace variable. This leads to the modified road model

$$G_{street} = G_{hp}G_{lp} = \frac{v_{Fzg}T_{hp}s}{(T_{hp}s + 1)(s + \beta v_{Fzg})} \quad (37)$$

which is applied to the full-car equations (11)–(13) instead of the standard road model. A comparison between the standard and the modified road model can be seen in Fig. 6 where it becomes apparent that both models differ in the frequency range below approximately 1 Hz and are identical for higher frequencies. The Kalman filter is then tuned using the adapted optimization routine and the modified street model.

Looking at the estimated absolute velocity of the body in Fig. 5 it can be seen that the low frequency disturbance is eliminated very well and only a small error between the reference and the estimated velocity remains.

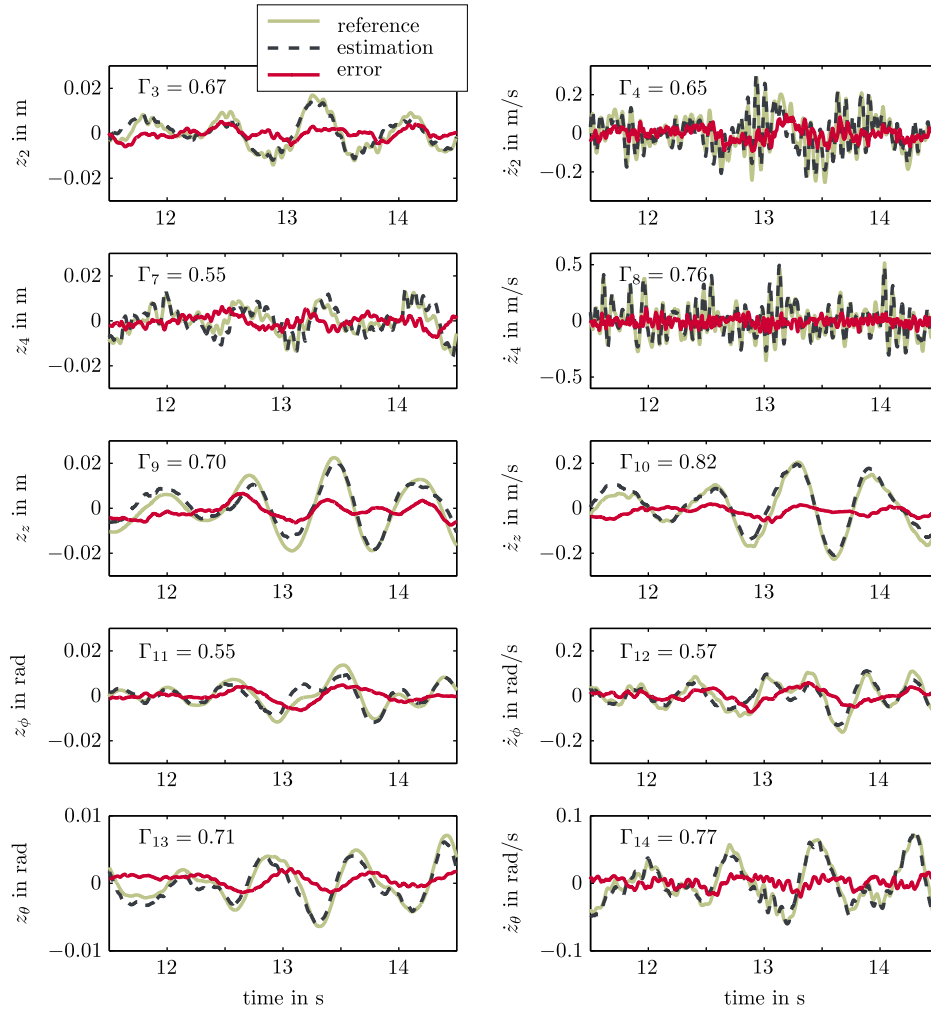


Fig. 7. Estimated and measured states of the full-car model including the fit values.

5. Experimental results

The observer and the two controllers have been implemented in an Audi Q5, which is by default equipped with four relative displacement sensors, a body heave accelerometer, two angular velocity sensors for the measurement of the pitch and roll velocity and a stock electronic control unit (ECU). This ECU has an Infineon TriCore processor and is programmed using the production code generator dSPACE TargetLink. It is noted that no adjustments have been made compared to the series production semi-active suspension system concerning the hardware or the software generation process. The skyhook-based series controller has been replaced by the model based approach, presented in this work, without increasing the computational power or the amount of sensors.

5.1. Observer validation

For the observer validation the experimental vehicle has been excited by a road profile on the 4-poster test rig. The reference signals are generated using four accelerometers mounted on the corners of the body and one accelerometer in the middle of each wheel by integration and highpass filtering methods. Fig. 7 shows a comparison between the reference signals and the estimated states

by the Kalman filter,² whereas the error between the signals is always very low indicating a high quality of estimation. The overall fit value

$$\Gamma_{\Sigma} = 1 - \frac{1}{14} \sum_{i=1}^{14} \frac{\|x_{i,ref} - \hat{x}_i\|_{rms}}{\|x_{i,ref}\|_{rms}} \quad (38)$$

for this road profile is 0.65, which is similar to the values given in Fröhlich (2007) for a real vehicle. This clearly shows that the observer including the modified road model operates very well in isolated conditions and even provides good estimates while ascending steep roads like it has been shown in Section 4.2. Therefore, the observer is used for LQ-based semi-active suspension control which is experimentally investigated in the next section.

5.2. Controller validation

The performance of the clipped optimal controller applied to the experimental vehicle is investigated on the 4-poster test rig (Fig. 3) using force sensors for measuring the dynamic tire loads and four accelerometers on the corners of the body for the measurement of the heave, pitch and roll accelerations. In order to evaluate the transfer behavior for the passive and the closed-

² The tire positions and tire velocities front right and rear left are not displayed for reasons of clarity.

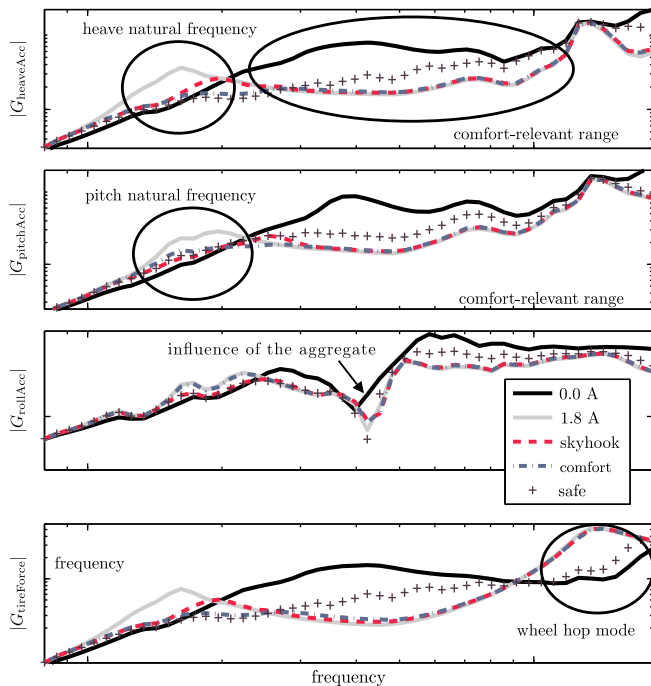


Fig. 8. Measured transfer behavior (scaling omitted).

loop configurations, the vehicle has been excited by sinus sweeps between 0.5 Hz and 25 Hz. Although the closed-loop system is nonlinear, the investigation of the frequency response is a common procedure in academic and industrial research for the performance evaluation of semi-active suspension systems (Savaresi, Poussot-Vassal, Spelta, Seneme, & Dugard, 2010). The input signals have been generated to excite the heave (all four hydraulic actuators move synchronously), pitch (the front actuators move in opposite direction to the rear ones) and roll (the left actuators move in opposite direction to the right ones) reaction of the vehicle. After a fast Fourier transform of the input and output signals, the transfer behavior from the road excitation ω to the body heave ($\ddot{z}_5$), roll ($\ddot{z}_6$) and pitch ($\ddot{z}_7$) accelerations as well as the tire force ($F_{z,dyn}$) are calculated.

The results are shown in Fig. 8 where it can be seen that both controllers decrease the amplitudes of the heave, pitch and roll accelerations over the whole frequency range. The controller *comfort* which has been optimized for ride comfort behaves like the softly damped passive suspension in the comfort-relevant range between 4 and 8 Hz (see Fig. 8, $|G_{heaveAcc}|$, $|G_{pitchAcc}|$). Looking at the body heave and pitch natural frequencies, it can be observed that the peaks of the passive soft suspension are reduced by the controller. Therefore, the body is well stabilized by simultaneously providing the highest possible comfort at high frequencies by the comfort orientated LQ-based controller. The performance is close to the results presented in Savaresi et al. (2010) where the optimal

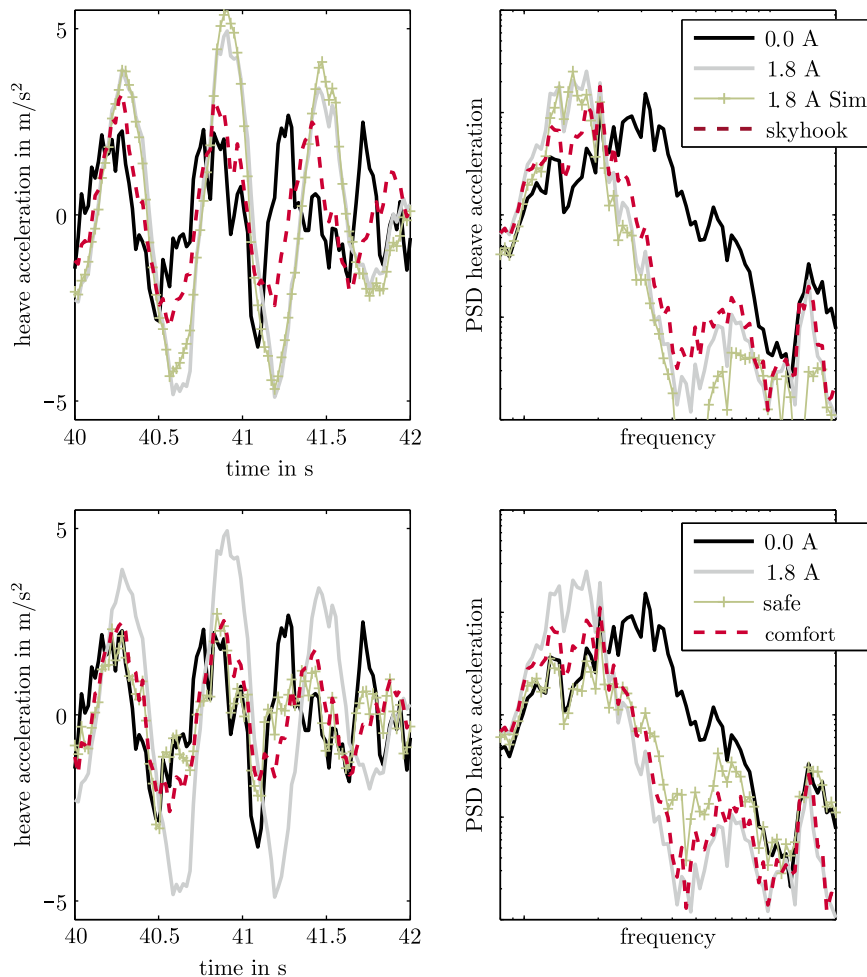


Fig. 9. Measured and simulated body acceleration for a real road profile.

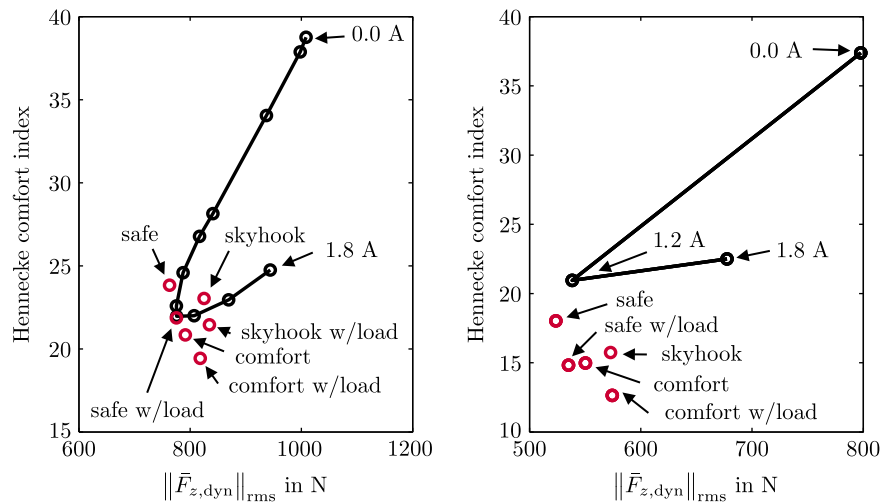


Fig. 10. Conflict diagrams for a road profile (left) and a sinus sweep (right).

frequency response for a semi-active suspension system has been derived using model predictive control. As the body should be isolated from leaning road surfaces in the roll direction, the roll transfer behavior (see Fig. 8, $|G_{rollAcc}|$) of the controller *comfort* is also satisfying because a lightly damped passive suspension behaves best for this usecase.

Comparing the LQ-based controller *comfort* to the also comfort orientated skyhook controller *skyhook*, the worse damping properties of the skyhook algorithm in the range of the heave natural frequency becomes apparent (see Fig. 8, $|G_{heaveAcc}|$, $|G_{pitchAcc}|$). An increasing skyhook damping factor worsens the isolation properties in the comfort-relevant range, whereas lower values lead to an even worse behavior for lower frequencies. Therefore, the trade-off between body stabilization and high frequency comfort is tighter for the skyhook controller than for the LQ-based controller.

The controller *safe* reduces the body movement at low frequencies and the tire force at the wheel hop mode considerably (see Fig. 8, $|G_{heaveAcc}|$, $|G_{pitchAcc}|$, $|G_{tireForce}|$). This behavior is beneficial for a braking maneuver because both effects have a bad influence on the dynamic tire load while braking. Especially the low frequency body movement causes huge tire load fluctuations leading to a significant extension of the braking distance what has been shown in Unger, Schimmack, et al. (2011). Considering the results of the study, a low rms value of the dynamic tire load stands for a low braking distance only if the damping of the body's natural frequency is high. As the low frequency damping of the controller *safe* is similar to the passive hard configuration, it cannot be improved any more. Therefore, it is possible to evaluate the rms value of the dynamic tire load for further investigations. In order to achieve this beneficial behavior with the controller *safe*, the amplitudes in the comfort-relevant range are increased slightly compared to the passive soft suspension (see Fig. 8, $|G_{heaveAcc}|$, $|G_{pitchAcc}|$, $|G_{tireForce}|$).

In addition to the analysis of the frequency response, which already contains rich information about the system behavior, the performance has been investigated using a real profile of a rough country road. The temporal responses and the power spectral densities of the body heave acceleration are shown in Fig. 9. In general, they show similar behavior as it has been observed for the excitation by sinusoidal disturbances. Looking at the temporal responses, it can be seen that neither the high amplitudes of the low frequency body movements of the passive soft setting nor the sharp acceleration peaks of the passive hard suspension can be observed for the controlled suspension configurations. The power spectral density plots indicate that the damping of the body's natural frequency is slightly worse for the controller *skyhook* than

it is achieved by the controller *comfort*. The LQ-based controller *dynamic* shows a well damped behavior at the body's natural frequency by slightly increased amplitudes at higher frequencies compared to the other controllers. In summary, the results, which had been discussed for the frequency response, can also be confirmed for the real road profile.

The comparison between the measurement and the simulation at $\lambda = 1.8$ A illustrates the validity of the nonlinear full-car model. Slight differences can be observed for higher frequencies, where the influences of unmodeled system dynamics, like bearing stiffnesses, increase.

Another performance evaluation is given by the conflict diagrams in Fig. 10, where the Hennecke comfort index and the mean rms value of the four dynamic tire loads $\bar{F}_{z,dyn}$ are shown for each configuration. The evaluation has been made for a real measured road profile and the sinus sweep input in the heave direction. For the road profile several passive configurations have been investigated leading to the black curve in Fig. 10 left and it can be seen that both controllers achieve a minimum in their respective performance aims. Especially for the sweep input, where a broad frequency range up to 25 Hz is considered, large benefits in comfort and safety of the controlled suspensions can be observed.

In order to investigate the behavior of the controllers in the case of a mass change of the body, the vehicle has been loaded with 75 kg at each front seat and 150 kg in the trunk resulting in an additional load of 300 kg. The results displayed in Fig. 10 for the controlled suspensions with additional load show a moderate increase in the tire load fluctuations, whereas the comfort index is decreased. As these effects can also be observed if any passive suspension is loaded the controlled suspension does not lead to unexpected behavior when the mass is varied during operation.

6. Conclusion

In this paper the experimental performance evaluation of an LQ-based semi-active observer and controller concept has been investigated. It has been shown that model-based control techniques can successfully be implemented in a real vehicle using series production hardware and offer a great performance potential.

Especially the parametrization of the controller has been addressed by minimizing suitable performance measures for an identified nonlinear full-car model and two controllers have been presented to enhance ride comfort and road holding.

Furthermore, the problem of low frequency disturbances which occur on real streets by leaning road surfaces, ascending or

descending hills or steady-state cornering causing corrupted estimates of the observer has been discussed. A new modified road model has been presented in order to compensate for these effects and it has been proven using measurements taken on a real road that the suggested method reliably filters the undesirable disturbances. The experimental validation of the observer on a 4-poster test rig has shown that all states are estimated properly. The investigation of a reduced sensors layout is proposed for future work.

Moreover, the performance of the controllers has been experimentally investigated and ride comfort improvements have been shown using a measure which considers the frequency weighted, stochastic heave and pitch motions of the car. The road holding has been taken into account by minimizing the braking distance leading to a controller which especially minimizes the vertical tire load at the body natural frequencies and the wheel hop mode. Finally it has been shown that a modified body mass does not lead to unexpected behavior of the controlled semi-active suspension system. Thereby, LQ-based semi-active suspension control is an appropriate choice for the implementation in series production vehicles.

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